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# Mass scale of new physics in the absence of the Higgs boson

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**Abstract.** We consider a hypothetical scenario in which the Higgs boson is absent, and attempt to constrain the mass scale  $\Lambda$  of the new physics that would take its place. Using recent measurements of  $\sin^2\theta_{\rm eff}^{\rm lept}$  and  $M_W$ , we show that, in a class of theories characterized by simple conditions, the upper bound on  $\Lambda$  is close to or smaller than the upper bound on  $M_H$  in the Standard Model, while in the complementary class  $\Lambda$  is not restricted by our considerations. The issue of fine-tuning when  $\Lambda$  is large is briefly discussed. Observations concerning the magnitude of the Higgs-boson contributions in the Standard Model are included. As a byproduct of our considerations, we discuss the usefulness and important properties of a radiative correction,  $\Delta r_{\rm eff}$ , that directly links  $\sin^2\theta_{\rm eff}^{\rm lept}$  with  $\alpha$ ,  $G_{\mu}$ , and  $M_Z$ .

## 1 Introduction

For a long time, the Higgs boson and the associated Higgs mechanism have been viewed as one of the most intriguing pillars of the Standard Model (SM). The consistency of this theory, its success in describing accurately a multitude of phenomena, and the emergence of Supersymmetry as a leading candidate for physics beyond the SM have given very strong support to the Higgs principle. Nonetheless, the provocative questions remain: What would happen if the Higgs boson was not found? Is it possible to surmise something of a general nature about the new physics that would take its place?

In this paper, we consider a hypothetical scenario in which the dynamical degrees of freedom associated with the Higgs boson are absent, and attempt to constrain the mass scale of the unknown new physics that would take its place, using current experimental information about two very sensitive parameters, namely  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$  and  $M_W$ .

# 2 Information from $\sin^2 \theta_{\rm eff}^{\rm lept}$

Our strategy is the following: we first consider a radiative correction  $\Delta r_{\rm eff}$  that directly links the accurately measured parameter  $\sin^2 \theta_{\rm eff}^{\rm lept}$ , employed to analyze the data at the  $Z^0$  peak, with  $\alpha$ ,  $G_{\mu}$ , and  $M_Z$ . Specifically, we have

$$s_{\text{eff}}^2 c_{\text{eff}}^2 = \frac{A^2}{M_Z^2 (1 - \Delta r_{\text{eff}})},$$
 (1)

where  $A^2=(\pi\alpha/\sqrt{2}G_\mu)$  and  $s_{\rm eff}^2=1-c_{\rm eff}^2$  is an abbreviation for  $\sin^2\theta_{\rm eff}^{\rm lept}$ .

An expression for  $\Delta r_{\rm eff}$  can be readily obtained by combining the relations

$$\hat{s}^2 \hat{c}^2 = \frac{A^2}{M_Z^2 (1 - \Delta \hat{r})},\tag{2}$$

where  $\hat{s}^2 = 1 - \hat{c}^2 = \sin^2 \hat{\theta}_w(M_Z)$  is the  $\overline{\text{MS}}$  electroweak mixing parameter [1–3] and

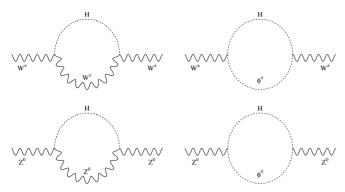
$$s_{\text{eff}}^2 = \text{Re}\,\hat{\kappa}_{\text{l}}(M_Z)\hat{s}^2,\tag{3}$$

studied in [4]. In (2),  $\Delta \hat{r}$  is the relevant radiative correction while, in (3),  $\hat{\kappa}_l(M_Z)$  is an electroweak form factor. Writing Re  $\hat{\kappa}_l(M_Z) = 1 + \Delta \hat{\kappa}_l$ , noting that  $\Delta \hat{\kappa}_l$  is numerically very small [4], and neglecting very small  $O(g^4)$  effects, one obtains (1) with the identification

$$\Delta r_{\text{eff}} = \Delta \hat{r} + \Delta \hat{\kappa}_{l} \frac{\alpha}{\hat{\alpha}} \left( 1 - \frac{\hat{s}^{2}}{\hat{c}^{2}} \right). \tag{4}$$

Note that the overall coupling of  $\Delta \hat{\kappa}_{\rm l}(\alpha/\hat{\alpha})$  is  $(\alpha/\pi \hat{s}^2)$ , in analogy with corresponding contributions to  $\Delta \hat{r}$  [1–3]. In the  $\overline{\rm MS}$  framework of [1–4],  $\hat{s}^2$  is evaluated at the scale  $\mu=M_Z$  and, therefore, the same applies to the radiative corrections  $\Delta \hat{r}$  and  $\Delta \hat{\kappa}_{\rm l}$ . However, we may consider the more general situation in which  $\hat{s}^2$ , and therefore also  $\Delta \hat{r}$  and  $\Delta \hat{\kappa}_{\rm l}$ , are evaluated at a general scale  $\mu$ . A very important property of (4) is that, in the SM, the  $\mu$  dependence of  $\Delta \hat{r}$  cancels against that of  $\Delta \hat{\kappa}_{\rm l}(\alpha/\hat{\alpha})(1-\hat{s}^2/\hat{c}^2)$ , so that  $\Delta r_{\rm eff}$  is scale independent. This also means that, if the  $\overline{\rm MS}$  pole subtractions in  $\Delta \hat{r}$  and  $\Delta \hat{\kappa}_{\rm l}(\alpha/\hat{\alpha})(1-\hat{s}^2/\hat{c}^2)$  are not implemented, their divergent parts cancel against each other. This property, which we have verified in the SM at the one-loop level, is to be expected on general grounds, since  $\Delta r_{\rm eff}$  is related, via (1), to physical observ-

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**Fig. 1.** Feynman diagrams contributing to  $(\Delta r_{\text{eff}})_H$  and  $\Delta r_H$  at one loop

ables. Equation (1) is analogous to the well-known expression

$$s^2 c^2 = \frac{A^2}{M_Z^2 (1 - \Delta r)},\tag{5}$$

where  $s^2 = 1 - c^2 = \sin^2 \theta_{\rm w} = 1 - M_W^2/M_Z^2$  is the electroweak mixing parameter in the on-shell scheme of renormalization,  $M_W$  and  $M_Z$  are the physical masses of the intermediate bosons, and  $\Delta r$  is the corresponding radiative correction [5].

For reasons that will become clear later on, in the discussion of new physics, we restrict the analysis to one-loop electroweak diagrams, but we include the  $O(\alpha \hat{\alpha}_s)$  corrections and the  $O(\alpha \hat{\alpha}_s^2)$  contributions to  $\Delta \rho$ . A convenient one-loop expression for  $\Delta \hat{r}$ , linear in the self-energies, is given in (15c) of [2], and the corresponding perturbative  $O(\alpha \hat{\alpha}_s)$  corrections can be obtained from [3]. The contributions to  $\Delta \hat{\kappa}_l$ , including QCD corrections, are given in [4]. For simplicity, we neglect very small effects involving light-fermion masses.

The next step in our strategy is to subtract from the SM quantity  $\Delta r_{\rm eff}$  the contribution from one-loop diagrams involving the Higgs boson, which constitute a gauge-invariant, albeit divergent subset [6]. They are depicted in Fig. 1. In the formulation of [2], this contribution is given by

$$(\Delta r_{\text{eff}})_{H} = \frac{\alpha}{4\pi \hat{s}^{2} \hat{c}^{2}} \left[ \left( \frac{5}{3} - \frac{3c^{2}}{2} \right) \left( \frac{1}{n-4} + C + \ln \frac{M_{Z}}{\mu} \right) + H(\xi) - \frac{3c^{2}}{4} \frac{\xi \ln \xi - c^{2} \ln c^{2}}{\xi - c^{2}} + \frac{19c^{2}}{24} + \frac{s^{2}}{6} \right],$$
(6)

where  $\xi = M_H^2/M_Z^2$ ,  $H(\xi)$  is a function studied in [5], n is the dimension of space-time,  $C = [\gamma - \ln(4\pi)]/2$  is the conventional constant accompanying the 1/(n-4) pole in dimensional regularization, and we explicitly exhibit the divergent and  $\mu$ -dependent contributions. We note that the last four terms in (6) coincide with the  $\overline{\text{MS}}$ -renormalized part of  $(\Delta r_{\text{eff}})_H$  when the scale  $\mu = M_Z$  is chosen. Calling their contribution  $(\Delta r_{\text{eff}})_H^{\overline{\text{MS}}}$  and subtracting (6) from  $\Delta r_{\text{eff}}$ , we have

$$\Delta r_{\text{eff}} - (\Delta r_{\text{eff}})_H = -\frac{\alpha}{4\pi \hat{s}^2 \hat{c}^2} \left( \frac{5}{3} - \frac{3c^2}{2} \right) \times \left( \frac{1}{n-4} + C + \ln \frac{M_Z}{\mu} \right) + \Delta r_{\text{eff}} - (\Delta r_{\text{eff}})_H^{\overline{\text{MS}}}, \tag{7}$$

where  $\Delta r_{\rm eff} - (\Delta r_{\rm eff})_{H}^{\overline{\rm MS}}$  is finite and independent of  $\mu$  and  $M_{H}$ . Thus, after subtracting the Higgs-boson diagrams, we are left with a divergent and scale-dependent expression! Next, we conjecture that contributions from unknown new physics cancel the divergence and scale dependence of (7). This contribution is then of the form

$$(\Delta r_{\text{eff}})_{\text{NP}} = \frac{\alpha}{4\pi \hat{s}^2 \hat{c}^2} \left(\frac{5}{3} - \frac{3c^2}{2}\right) \left(\frac{1}{n-4} + C + \ln\frac{M}{\mu}\right),$$
(8)

where  $\ln(M/\mu)$  represents the new-physics contribution at scale  $\mu$ , in the  $\overline{\rm MS}$  scheme. It is related to the scale  $\Lambda$  of the new theory by

$$\ln \frac{M}{\mu} = \ln \frac{\Lambda}{\mu} + K,$$
(9)

where  $K = \ln(M/\Lambda)$  is the new-physics contribution at scale  $\Lambda$ . Adding (8) to (7), we find that, in the new scenario (NS) in which new-physics effects take the place of the Higgs-boson contributions, the SM parameter  $\Delta r_{\rm eff}$  is replaced by

$$(\Delta r_{\text{eff}})_{\text{NS}} = \frac{\alpha}{4\pi \hat{s}^2 \hat{c}^2} \left( \frac{5}{3} - \frac{3c^2}{2} \right) \ln \frac{M}{M_Z} + \Delta r_{\text{eff}} - (\Delta r_{\text{eff}})_H^{\overline{\text{MS}}}.$$
(10)

The first term represents the new-physics contribution at scale  $M_Z$ . As a check, we may consider the particular case in which the "new physics" is provided by the SM Higgs boson. For large  $M_H$ , the leading one-loop Higgs-boson contribution to  $\Delta r_{\rm eff}$  in the SM is given by  $(\alpha/4\pi\hat{s}^2\hat{c}^2)$   $(5/3-3c^2/2)\ln(M_H/M_Z)$ , so that, in the asymptotic limit, M would then be identified with  $M_H$ .

Next, we constrain  $\ln(M/M_Z)$  from present experimental information. Constraints on  $\Lambda$  will be discussed later on. Inserting the current central value  $s_{\rm eff}^2=0.23151\pm0.00017$  [7] into (1), we find the experimental result  $(\Delta r_{\rm eff})_{\rm exp}=0.06053\pm0.00049$ . As  $\hat{s}^2$  is numerically very close to  $s_{\rm eff}^2$  [4], in the evaluation of (10) we substitute  $\hat{s}^2\to s_{\rm eff}^2=0.23151,~\hat{c}^2\to c_{\rm eff}^2$ , while for  $s^2=1-c^2$  we employ 0.22223, corresponding to the current central values,  $M_W=80.419~{\rm GeV}$  and  $M_Z=91.1871~{\rm GeV}$  [7]. Using  $M_t=(174.3\pm5.1)~{\rm GeV},~\hat{\alpha}_{\rm s}(M_Z)=0.119\pm0.002$  [7], and  $\Delta\alpha_h^{(5)}=0.02804\pm0.00065$  [8], (10) leads to

$$(\Delta r_{\text{eff}})_{\text{NS}} = \frac{\alpha}{4\pi s_{\text{eff}}^2 c_{\text{eff}}^2} \left(\frac{5}{3} - \frac{3c^2}{2}\right) \ln \frac{M}{M_Z} + 0.06049$$
$$\pm 0.00065 \pm 0.00051 \pm 0.00002. \tag{11}$$

The quantity  $\Delta \alpha_h^{(5)}$  stands for the contribution of the five light quark flavors (u,d,s,c,b) to the renormalized photon vacuum-polarization function evaluated at  $q^2 = M_Z^2$ .

Later on, we illustrate the corresponding results for the value  $\Delta \alpha_h^{(5)} = 0.02770 \pm 0.00016$  [9], determined by applying perturbative QCD down to energies of a few GeV. The errors in (11) are induced by those in  $\Delta \alpha_h^{(5)}$ ,  $M_t$ , and  $\hat{\alpha}_{\rm s}(M_Z)$ , respectively. The last two are found by independently varying  $M_t$  and  $\hat{\alpha}_{\rm s}(M_Z)$  in the evaluation of (10). Comparing (11) with  $(\Delta r_{\rm eff})_{\rm exp}$  and combining the errors in quadrature, we obtain

$$\ln \frac{M}{M_Z} = 0.028 \pm 0.586,$$
(12)

corresponding to the central value  $M_{\rm c}=94$  GeV and an upper bound  $M_{95}=245$  GeV at the 95% confidence level (CL). We note that we have employed QCD corrections to  $\Delta\rho$  in the conventional  $M_t$  approach [10], which, in calculations restricted to one-loop electroweak amplitudes, leads to larger values of  $\ln(M/M_Z)$  than optimized methods [11]. Thus, as far as uncertainties in the QCD corrections are concerned, we take (12) to be a rather conservative estimate.

## 3 Information from $M_W$

Turning our attention to  $\Delta r$ , we employ (21) of [2] with the approximation  $(c^2/s^2)\Delta\hat{\rho}(1-\Delta\hat{r}_{\rm w})\approx (c^2/s^2)\Delta\hat{\rho}(\alpha/\hat{\alpha})$ , where  $\Delta\hat{\rho}$  and  $\Delta\hat{r}_{\rm w}$  are radiative corrections defined in that work. In this way, we obtain an expression for  $\Delta r$  that depends linearly on the self-energies. In the SM, this approximation neglects two-loop effects of order  $2\times 10^{-4}$ . The SM Higgs-boson contribution to  $\Delta r$  in the SM is given by

$$\Delta r_H = \frac{\alpha}{4\pi \hat{s}^2} \left[ \frac{11}{6} \left( \frac{1}{n-4} + C + \ln \frac{M_Z}{\mu} \right) + \frac{H(\xi) - H(\xi/c^2)(1 - 2s^2)}{s^2} - \frac{3\xi}{4(\xi - c^2)} \right] \times \ln \frac{\xi}{c^2} + \frac{23}{24} + \frac{\ln c^2}{6} \left( \frac{1}{2} - \frac{5c^2}{s^2} \right) .$$
 (13)

The relevant Feynman diagrams are depicted in Fig. 1. The expression analogous to (10) becomes

$$\Delta r_{\rm NS} = \frac{11\alpha}{24\pi\hat{s}^2} \ln \frac{M'}{M_Z} + \Delta r - \Delta r_H^{\overline{\rm MS}},\tag{14}$$

where the first term represents the new-physics contribution at scale  $M_Z$  and  $\Delta r$  is the SM correction. Employing the same input parameters as in (11), (14) leads to

$$\Delta r_{\rm NS} = \frac{11\alpha}{24\pi s_{\rm eff}^2} \ln \frac{M'}{M_Z} + 0.03355 \pm 0.00065$$
$$\pm 0.00191 \pm 0.00007. \tag{15}$$

Inserting  $M_W = (80.419 \pm 0.038)$  GeV and  $M_Z = (91.1871 \pm 0.0021)$  GeV [7] into (5), we obtain the experimental value  $\Delta r_{\rm exp} = 0.03298 \pm 0.00230$ . Comparison of (15) with  $\Delta r_{\rm exp}$  leads to

$$\ln \frac{M'}{M_Z} = -0.124 \pm 0.666,\tag{16}$$

corresponding to  $M_{\rm c}'=81$  GeV and  $M_{95}'=240$  GeV. We note that  $M_{\rm c}'$  and  $M_{95}'$  are very close to  $M_{\rm c}$  and  $M_{95}$ , respectively.

#### 4 Discussion and conclusions

It is interesting to compare the above results for M and M' with SM estimates of  $M_H$ . Using the same input parameters leading to (12), we apply Eq. (3) of [12] in the  $\overline{\rm MS}$  scheme discussed in that paper to extract  $\ln(M_H/M_Z)$  from  $s_{\rm eff}^2$  in the SM, with the result

$$\ln \frac{M_H}{M_Z} = 0.016 \pm 0.629,$$
(17)

which corresponds to a central value  $M_H^c = 93$  GeV and a 95% CL upper bound  $M_H^{95} = 260$  GeV. If, instead,  $M_W$  is employed as the basic input, using (4) of [12] in the  $\overline{\rm MS}$  scheme, we find in the SM

$$\ln \frac{M_H}{M_Z} = -0.728 + 0.972 -1.596, \tag{18}$$

 $M_H^c = 44$  GeV and  $M_H^{95} = 193$  GeV. We see that (17), the SM determination of  $M_H$  derived from  $s_{\rm eff}^2$ , is very close to (12), the M estimate from the same source. On the other hand, the SM estimate of  $M_H$  derived from  $M_W$  [(18)] is somewhat more restrictive than the corresponding M' determination [(16)], with  $M_H^c$  and  $M_H^{95}$  lower by about 37 GeV and 47 GeV, respectively. Nonetheless, the overall picture that emerges is that the M and M' estimates are very close and also quite similar to the SM determinations of  $M_H$ .

The same pattern is apparent when one employs the "theory-driven" determination of  $\Delta\alpha_h^{(5)}$ , yielding  $\Delta\alpha_h^{(5)}=0.02770\pm0.00016$  [9]. In this case, using  $s_{\rm eff}^2$  as input, we find  $M_{\rm c}=116$  GeV and  $M_{95}=239$  GeV, while the SM values for  $M_H$  are  $M_H^{\rm c}=116$  GeV and  $M_H^{95}=249$  GeV. If, instead,  $M_W$  is used as input, we obtain  $M_c'=87$  GeV and  $M_{95}'=252$  GeV, while the SM values are  $M_H^{\rm c}=51$  GeV and  $M_H^{95}=205$  GeV. We also note that the central values obtained with this determination of  $\Delta\alpha_h^{(5)}$  are somewhat larger than in the case of  $\Delta\alpha_h^{(5)}=0.02804\pm0.00065$  [8], but the 95% CL upper bounds are generally close.

The closeness of M, M', and the corresponding  $M_H$  estimates in the SM is perhaps surprising. We have already pointed out, in the discussion after (10), that at the one-loop level one expects M and M' to approach asymptotically the SM estimate for  $M_H$ . However, in the range 100 GeV  $\lesssim M_H \lesssim 300$  GeV, we are far away from the asymptotic domain. Furthermore, there are important differences between our approach to evaluate M and M', and the SM calculation of  $M_H$ . In the latter, one considers the detailed  $M_H$  dependence of the one-loop corrections, and, moreover, current studies include complicated two-loop  $M_H$ -dependent effects of  $O(\alpha^2 M_t^4/M_W^4)$  and  $O(\alpha^2 M_t^2/M_W^2)$  [13]. It is known that, in the SM, these two-loop effects decrease significantly the derived value

for  $M_H$  and its upper bound. In the hypothetical scenario we consider in this paper, in which the degrees of freedom associated with the Higgs boson are absent, it is not possible to incorporate such higher-order  $M_H$ -dependent effects, and, for that reason, we have restricted ourselves to one-loop electroweak diagrams. Nonetheless, the results of the two approaches are very similar. In particular, it is quite remarkable that the simple formula of (11) with the replacement  $M \to M_H$  reproduces very accurately the SM calculation for  $M_H \approx 100$  GeV and remains a reasonable approximation for larger values of  $M_H$ . For instance, for  $M_H = 100, 300, 600 \text{ GeV}$ , the differences between (11) with  $M \to M_H$  and the SM calculations of [12] amount to  $-1 \times 10^{-5}$ ,  $1.6 \times 10^{-4}$ , and  $2.7 \times 10^{-4}$  (corresponding to differences in  $s_{\rm eff}^2$  of less than  $1 \times 10^{-5}$ ,  $6 \times 10^{-5}$ , and  $1\times10^{-4}$ ), respectively. Over the same  $M_H$  range, (15) with  $M' \to M_H$  leads to differences in  $M_W$  of 23 to 10 MeV, relative to the SM calculation of [12].

We now discuss the implications of these results for the scale  $\Lambda$  of the unknown, alternative theory. Following (9), we write

$$\ln \frac{M}{M_Z} = \ln \frac{\Lambda}{M_Z} + K \tag{19}$$

in the case of  $\Delta r_{\text{eff}}$  [(10)–(12)], and

$$\ln \frac{M'}{M_Z} = \ln \frac{\Lambda}{M_Z} + K' \tag{20}$$

in the case of  $\Delta r$  [(14)–(16)]. As the alternative theory is not specified, K and K' are unknown constants. Equations (19) and (20) permit us to classify all alternative theories into two broad categories: (1) theories in which either Kor K' (or both) are positive; (2) theories in which both Kand K' are negative. We see that theories of the first class are bounded by one of the two equations. If K is positive, at 95% CL we have  $\Lambda \lesssim 245$  GeV [(12)]. If K' is positive,  $\Lambda \lesssim 240 \text{ GeV } [(16)]$ , independently of the precise value of |K|. We note that these limits are approximately of the same magnitude as those currently derived for  $M_H$  in the SM. It is worth noting that  $K' - K = -0.152 \pm 0.887$ , so that one can entertain the possibility that both K and K'are simultaneously small, which may occur in a subclass among weakly interacting theories. If  $|K|, |K'| \leq 0.1$ , for instance, the  $\Lambda$  bounds would be quite close to the SM bounds on  $M_H$ .

Theories of the second class evade our bounds and may correspond to large values of  $\Lambda$ , as in the scenario recently discussed in [14]. If  $\Lambda=1$  TeV, for instance, we have  $K=-2.367\pm0.586$  [(12) and (19)] and  $K'=-2.519\pm0.666$  [(16) and (20)]. Thus, for  $\Lambda=1$  TeV, in order to reproduce our central values, a fine-tuning of the parameters K and K' is necessary. However, if K and K' differ from their central values by about one sigma, one may argue that no excessive fine-tuning is necessary. On the other hand, even at the one-sigma level, a substantial cancellation between the logarithmic and constant terms is required in order to accomodate our results, an observation that may become sharper in the future as errors decrease.

It is important to emphasize that in our formulation we do not assume ab initio that the scale  $\Lambda$  of the new physics

is large relative to  $M_Z$ . In fact, our procedure has been to focus on the radiative corrections associated with two very precise observables, subtract exactly the Higgs-boson contribution, and then restrict  $\Lambda$  from accurate experiments. This leads to our conclusion that K>0 and/or K'>0 is a sufficient condition for  $\Lambda$  to have low 95% CL upper bounds. The usefulness of this condition stems from the fact that, if satisfied, one can immediately draw the conclusion that  $\Lambda$  is sharply restricted. In some alternative formulations, such as those based on the S, T, and U parameters, one assumes ab initio that  $\Lambda$  is large relative to  $M_Z$ . Although such formulations permit to discuss the large- $\Lambda$  case, they are clearly not suitable to draw conclusions or derive sufficiency conditions in the case when  $\Lambda$  is close to the electroweak scale.

We also note that the sharp constraints derived above for theories of the first class are based on the smallness of  $\ln(M/M_Z)$  in (12) and  $\ln(M'/M_Z)$  in (16). In turn, this is related to the very curious fact that the SM Higgsboson contributions to  $(\Delta r_{\rm eff})_H^{\overline{\rm MS}}$  and  $\Delta r_H^{\overline{\rm MS}}$  are highly suppressed for current experimental inputs. In order to show this, it is sufficient to point out that  $(\alpha/4\pi s_{\rm eff}^2 c_{\rm eff}^2)$   $(5/3-3c^2/2) \ln(M/M_Z)$  in (11) and  $(11\alpha/24\pi s_{\rm eff}^2) \ln(M'/M_Z)$  $M_Z$ ) in (15) replace the full one-loop Higgs-boson contributions, as well as the  $M_H$ -dependent effects of  $O(\alpha^2 M_t^4/M_W^4)$  and  $O(\alpha^2 M_t^2/M_W^2)$ , currently incorporated in the calculation of these basic corrections [12,13]. Using the values of  $\ln(M/M_Z)$  in (12) and  $\ln(M'/M_Z)$ in (16), we find that these contributions amount to only  $(0.46 \pm 9.57) \times 10^{-4}$  and  $(-5.69 \pm 30.61) \times 10^{-4}$ , respectively. As the typical order of magnitude of electroweak corrections is  $\alpha/(\pi s_{\rm eff}^2) \approx 0.01$ , we conclude that the SM Higgs-boson contributions to  $(\Delta r_{\rm eff})_H^{\overline{\rm MS}}$  and  $\Delta r_H^{\overline{\rm MS}}$  are, indeed, highly suppressed. This corresponds to the small values of  $\ln(M/M_Z)$  and  $\ln(M'/M_Z)$  found above, and leads to the sharp constraints on theories of the first class. As an illustrative counter-example, if the SM Higgs-boson contributions to these corrections were 0.5% for current experimental inputs, and the errors were the same, we would obtain  $\ln(M/M_Z) = 3.064 \pm 0.586$  and  $\ln(M'/M_Z) =$  $1.087 \pm 0.666$ , corresponding to  $M_{95} = 5.10$  TeV and  $M'_{95} = 806$  GeV, and the constraints on theories of the first class would be irretrievably lost.

The simplest mechanism that gives positive contributions to K and K' is a heavy, ordinary, degenerate fermion doublet. It contributes 2/11 to K' [15] and, to very good approximation, 1/3 to K. Theories that contain several degenerate doublets, such as the simplest technicolor models, are likely to belong to the first class. Instead, heavy non-degenerate doublets with sufficiently large mass splittings contribute negatively to K and K' and are likely to lead to second-class theories.

In this paper, we have considered the constraints derived from the two most sensitive measurements, to wit those of  $\sin^2\theta_{\rm eff}^{\rm lept}$  and  $M_W$ . Additional information can be obtained from other observables, such as the Z-boson width. However, the corresponding effective logarithm is not expected to be as tightly constrained as those in (12) and (16).

In summary, there are very powerful arguments to expect the discovery of the Higgs boson. In the unlikely event that this fundamental particle is not found, one expects the emergence of new physics. We have seen that in a class of alternative theories, characterized by a simple condition, the scale  $\Lambda$  is bounded by limits that are roughly of the same magnitude as those currently derived for  $M_H$  in the SM, while the complementary class is not restricted by our considerations. As a by-product, we have emphasized the usefulness of a basic electroweak correction,  $\Delta r_{\rm eff}$ , that directly links  $s_{\rm eff}^2$ ,  $\alpha$ ,  $G_{\mu}$ , and  $M_Z$ .

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